Developing Measures of Inductive Reasoning Using Logic-Based Measurement
- Tutorial -

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Who we are: US Customs and Border Protection in DHS

- US Customs and Border Protection (CBP)
  - America's unified border agency
  - Twin goals of anti-terrorism and facilitating legitimate trade and travel
  - Secures 314 ports of entry into US and borders between ports
  - Prevents narcotics, agricultural pests, smuggled goods and inadmissible visitors (e.g., aliens with outstanding criminal warrants) from entering US
- Personnel Research and Assessment Division (PRAD)
  - Part of CBP's Office of Human Resources Management
  - Group of I/O psychologists who design, develop, validate, and implement wide range of competency-based assessments
    - Entry-level and promotional assessments
  - Serves CBP, US Immigrations and Customs Enforcement and US Citizenship and Immigration Services

Job-Related Reasoning Skills

DHS employees use their reasoning skills in countless decisions, determinations, and investigations:

- A Border Patrol Agent deciding whether an impending confrontation involves dangerous individuals
- a Customs and Border Protection Officer determining if an alien should be admitted
- a supervisory Special Agent deciding whether to open a case based on certain evidence

Reasoning skills are used in:

- applying rules, making determinations, making predictions, and in problem solving, on-the-spot decision making, and complex, deliberate decision making.

Why Should Logic-Based Reasoning Questions Be Used in Selection Tests?

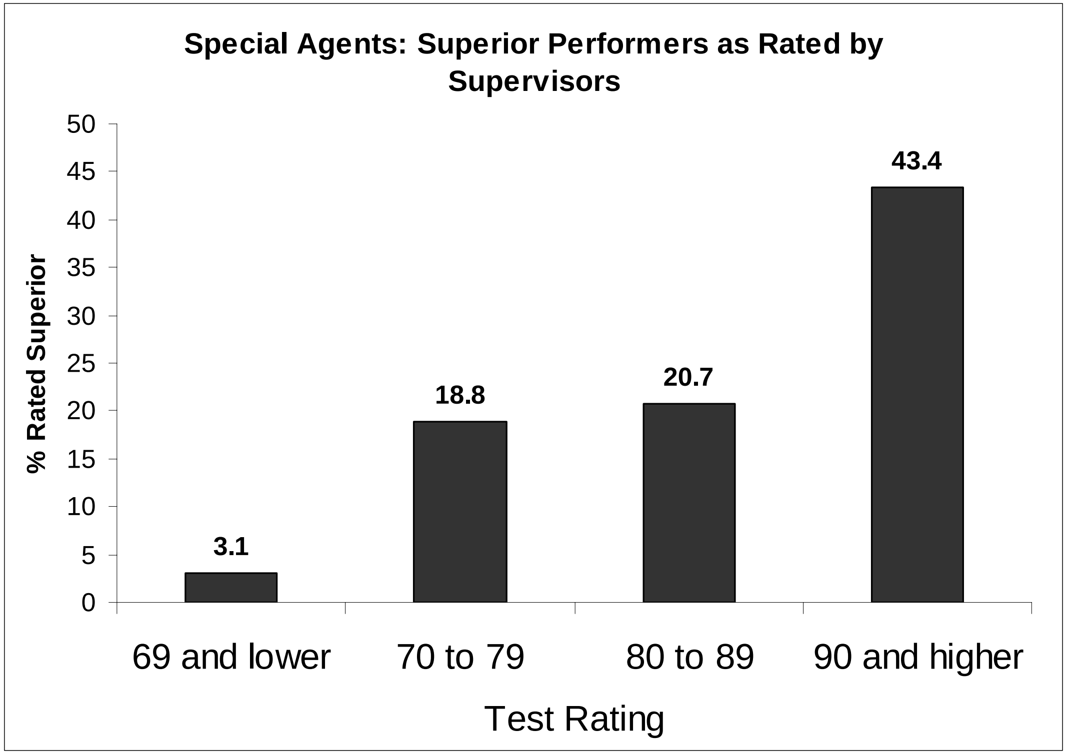
- Logic-based questions measure Reasoning, which is a well-established construct in the psychometric literature
  - Reasoning can be broadly defined as drawing inferences from information.
- Reasoning skills are among most important job skills
- Logical formulas define the content domain of reasoning
- LBM questions replicate the logical formulas that are used on the job – content validity evidence.

Why Should Logic-Based Reasoning Questions Be Used in Selection Tests?

- LBM questions have proven to be excellent predictors of training success and job performance.
- Average validities (Hayes et al., 2003):
  - training r = .60, lcv = .6
  - work simulation r = .60, lcv = .6
  - supervisory ratings r = .27, lcv = .2

Why Should Logic-Based Reasoning Questions Be Used in Selection Tests?

High Scorers Excel on the Job



Why Should Logic-Based Reasoning Questions Be Used in Selection Tests?

- LBM questions always have excellent psychometric statistics (item analysis)

- Because questions almost always “work,” you do not need to write lots of extra items.

Getting Started in Using Logic-Based Reasoning Questions

Item writers must spend time becoming familiar with basic principles of logic.

- This tutorial will begin the process of familiarization.

After becoming familiar with logic, the next step is to create or adapt a taxonomy of logical formulas.

- A taxonomy defines the content domain of the reasoning construct, both for the job and for the selection test
- A taxonomy will be provided in this tutorial

How is Inductive Reasoning Different from Deductive Reasoning?

Logicians are concerned with formulating rules for drawing correct inferences.

Logicians have defined two types of reasoning: deductive and inductive.

We use the definitions that logicians use:

- Inductive reasoning is reasoning with incomplete information and drawing a conclusion that may not be true, but which has some probability of being true.
- Deductive reasoning is reasoning with complete information to draw a certain or necessary conclusion.

How is Inductive Reasoning Different from Deductive Reasoning?

Historical note on Induction in psychometrics:

In early psychometric research, induction was defined as reasoning from the particular to general or discovering a rule.

- Thurstone, 1938, Primary Mental Abilities study:

Induction was defined as that characteristic of tests which requires subjects "to find a rule or principle for each item in the test."

- This incomplete definition persisted into the late 20th century. In Carroll's 1993 compendium, Human Cognitive Abilities, inductive tasks were defined as those in which the subject is required "to inspect a set of materials and… "induce a rule governing the materials…"

- Typical questions used for measuring induction as thus defined are letter series and figure analogies.

For example: Letter series

Choose the answer that continues the following series:

b n c d n e f g n h l j k

How is Inductive Reasoning Different from Deductive Reasoning?

Inductive reasoning

- occurs in the absence of complete information
- leads to valid conclusions that are not **necessarily** true, but only have some probability of being true

Deductive reasoning

- occurs in the presence of complete information
- leads to valid conclusions that are **necessarily** true, if the evidence is true

How is Inductive Reasoning Different from Deductive Reasoning?

Inductive reasoning

- The evidence does not guarantee the truth of the conclusion, but it gives us a good reason to believe in the truth of the conclusion. The premises support the conclusion.

Deductive reasoning

- The truth of the evidence makes the truth of the conclusion certain.

How are Inductive Questions Different from Deductive Questions?

Example of deductive schemas

- *Premise:* Of all first-line supervisors in the agency, 95% have taken the basic supervisory course.

Conclusions:

- Nearly all first-line supervisors in the agency have taken the basic supervisory course.
- Some people who have taken the basic supervisory course are first-line supervisors in the agency.

How are Inductive Questions Different from Deductive Questions?

Example of inductive schemas

- *Premise:* Of all first-line supervisors in the agency, 95% have taken the basic supervisory course.
- Pat is one of the first-line supervisors.
- *Conclusion:*
- It is very likely that Pat has taken the basic supervisory course.

Criteria for Correct Induction

- The degree of probability claimed for the conclusion must be supported by the premises.
  - Example of conclusion not supported by the premises:

It is very unlikely that Pat has taken the basic supervisory course.

Criteria for Correct Induction

- The total available evidence must be used in forming a conclusion.
- If you obtain additional evidence, you may need to revise your conclusion.
  - Example of additional evidence:
  - "Supervisors in Pat's sector have attended the basic supervisory course in lower-than-average proportions, because of other urgent initiatives in the last year."
  - How would you revise the conclusion about Pat's attendance at supervisory training?

The Meaning of Probability

- Probability is
  - the likelihood that a conclusion is true, given certain evidence.
  - the *degree of confirmation* for the conclusion provided by the premises. This is the common interpretation of probability in inductive logic.

The Meaning of Probability

- The word probability can also be used for *relative frequency* -- how often an event occurs relative to other possible events.
- Example about first-level supervisors in the agency:
  - 95 out of 100 of these supervisors had taken the basic course in supervision. The relative frequency of 95/100 provided our best estimate of the probability that any one first-level supervisor had attended the training.

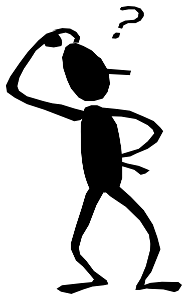
The Meaning of Probability

- Probabilities vary between 0 and 1.
  - A value of 1 represents absolute confirmation -- a condition that applies only to deductive conclusions.
  - A value of 0 represents absolute exclusion -- a condition that applies only to deductive conclusions.
  - For values between 0 and 1, numerical values may or may not be assigned.
    - Example: the word “probably” means “more likely than not” or “with a probability greater than .5.”
    - In test questions, we commonly use the expression “ there is a 95% chance” rather than “with a .95 probability.”

Expressions of Probability

• Probability greater than .7	greater than a 70% chance
• Probability equal to or greater than .7	at least a 70% chance no less than a 70% chance
• Probability equal to .7	exactly a 70% chance
• Probability equal to or less than .7	up to a 70% chance no greater than a 70% chance no more than a 70% chance
• Probability less than .7	less than a 70% chance
• Probably	greater than a 50% chance more likely than not

LBM QUESTION

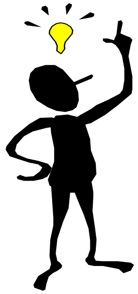


The printed output of some computer-driven printers can be recognized by forensic analysts. The “Acme M200” printer was manufactured using two different inking mechanisms, one of which yields a “Type A” micoropattern of ink spray. Of all Acme M200 printers, 70% produce this Type A micropattern, which is also characteristic of some models of other printers such as the Baleford 200, of which 60% produce the Type A micropattern. Forensic analysts at a crime lab have been examining a kidnap ransom note which was produced by the Acme M200 printer.

From the information given above, it can be validly concluded that this note

A) has the Type A micropattern, with a probability of .3
B) has the Type A micropattern, with a probability of .7
C) does not have the Type A micropattern, with a probability of .7
D) may not have the Type A micropattern, but the probability cannot be determined
E) may have the Type A micropattern, but the probability cannot be determined

LBM QUESTION



Of all Acme M200 printers, 70% produce this Type A micropattern, which is also characteristic of some models of other printers.

Forensic analysts at a crime lab have been examining a kidnap ransom note which was produced by the Acme M200 printer.

From the information given above, it can be validly concluded that

B) this note has the Type A micropattern, with a probability of .7

Getting Started

Learn to diagram a sentence logically.

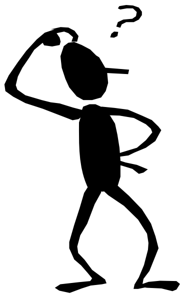
Four Parts of a Statement

- Quantifier - All, Some, 95%, one-third
- Subject term - noun
- Verb - to be
- Predicate term - noun, adjective, adjectival phrase or clause (that which is affirmed or denied of the subject)

Statement: Five percent of detainees are arrested.

Parts: Q S V P

EXERCISE



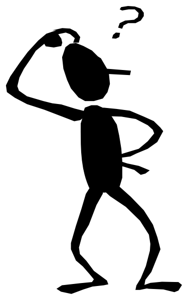
Sentence: Eighty percent of the desks in the office are new.

Logical Statement: Of all S, (m/n) are P

Logical Parts:

1. Quantifier -
2. Subject Term -
3. Verb -
4. Predicate Term -

EXERCISE



Sentence: Eighty percent of the desks in the office are new.

Logical Statement: Of all S, (m/n) are P

Logical Parts:

1. Quantifier – m/n = .80
2. Subject Term – desks in the office
3. Verb - are
4. Predicate Term – new things

Quantifier

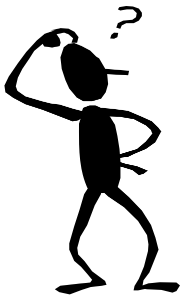
We use “m/n” to denote the numeric quantifier in inductive items.

In the previous example m/n = .80

The numerator ‘n’ is the total number of individuals in set S who are also members of set P.

The denominator ‘m’ is the total number of individuals known to be members of set S.

EXERCISE



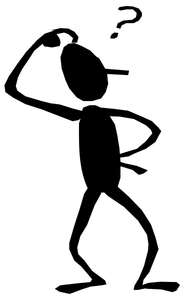
Sentence: Fifteen percent of the detainees are repeat offenders.

Logical Statement: Of all S, (m/n) are P

Logical Parts:

1. Quantifier -
2. Subject Term -
3. Verb -
4. Predicate Term -

EXERCISE



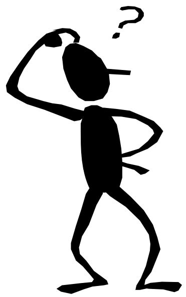
Sentence: Fifteen percent of the detainees are repeat offenders.

Logical Statement: Of all S, (m/n) are P

Logical Parts:

1. Quantifier – m/n = .15
2. Subject Term – detainees
3. Verb - are
4. Predicate Term – repeat offenders

EXERCISE



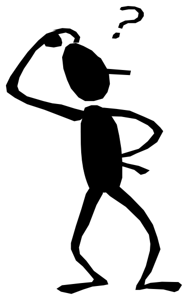
Sentence: Ninety percent of the firefighters are not supervisors.

Logical Statement: Of all S, (m/n) are not P

Logical Parts:

1. Quantifier -
2. Subject Term -
3. Verb -
4. Predicate Term -

EXERCISE



Sentence: Ninety percent of the firefighters are not supervisors.

Logical Statement: Of all S, (m/n) are not P

Logical Parts:

1. Quantifier – m/n = .90
2. Subject Term - firefighters
3. Verb – are not
4. Predicate Term - supervisors

LBM QUESTION

Of all Acme M200 printers, 70% produce this Type A micropattern, which is also characteristic of some models of other printers.

Q = m/n = 70%

S = Acme M200 printers

P = printers that produce the Type A micropattern

Forensic analysts at a crime lab have been examining a kidnap ransom note which was produced by the Acme M200 printer.

a = the kidnap ransom note

Of all S, (m/n) are P. *a* is an S.

LBM Question

A) this note has the Type A micropattern, with a probability of .3
a is a P, with a probability of (1-m/n)

B) this note has the Type A micropattern, with a probability of .7
a is a P, with a probability of (m/n)

C) this note does not have the Type A micropattern, with a probability of .7
a is not a P, with a probability of (1-m/n)

D) this note may not have the Type A micropattern, but the probability cannot be determined
a is not a P, with an unknown probability

E) this note may have the Type A micropattern, but the probability cannot be determined
a is a P, with an unknown probability

Negating Terms

- *To negate a term* is to alter a term so that the altered term does not refer to the same set of things to which the unaltered term refers.

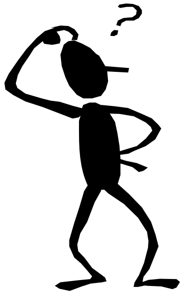
- The set of things to which the original term refers and the set of things to which the negated term refers have NO members in common.

Negating Terms

Examples:

- the negated set of "combatants" is "noncombatants”
- the negated set of "attainable goals" is the set "unattainable goals”
- the negated set of “logic textbooks” is “textbooks other than logic textbooks”

Negating Terms



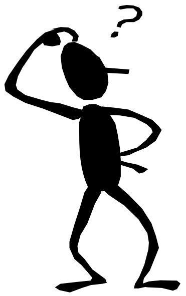
Exercise:

Statement: Ninety-five percent of the soldiers are armed.

Negated subject term:

Negated predicate term:

Negating Terms



Exercise:

Statement: Ninety-five percent of the soldiers are armed.

Negated subject term: civilians

Negated predicate term: unarmed individuals

Using the Taxonomy

Table B:

B Premises Of all S, m/n are P. *a* is an S.

Valid Conclusions

B1 with a prob. of m/n, *a* is a P

B2 with a prob. of 1 - m/n, *a* is not a P

B3 with a prob. of m/n, *a* is not a non-P

B4 with a prob. of 1 - m/n, *a* is a non-P

Using the Taxonomy

Table B:

B Premises Of all S, m/n are P. *a* is an S.

Invalid Conclusions

B5 with an unknown prob., *a* is a P

B6 with an unknown prob., *a* is not a P

B7 with a prob. of m/n, *a* is not a P

B8 with a prob. of 1 – m/n, *a* is a P

B9 with an unknown prob., *a* is not a non-P

B10 with an unknown prob., *a* is not a P

B11 with a prob. of m/n, *a* is a non-P

B12 with a prob. of 1 – m/n, *a* is not a non-P

Using the Taxonomy

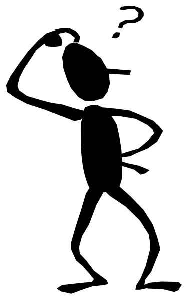
- Building an LBM question with valid and invalid conclusions
- Steps
  1. Choose a statement(s) for the premise(s)
  2. Go to the table in the taxonomy that serves your premise
  3. Choose one valid conclusion
  4. Choose invalid conclusions

Using the Taxonomy

Premises: Ninety-three percent of the confiscated visas are genuine. Ms. Greene's visa is one of the confiscated visas.

- Valid Conclusion:
  - B4 with a probability of .07, Ms. Greene's visa is fraudulent
- Invalid Conclusions:
  - B7 with a probability of .93, Ms. Greene's visa is not genuine
  - B9 Ms. Greene's visa may not be fraudulent, but the probability cannot be determined
  - B12 with a probability of .07, Ms. Greene's visa is not fraudulent

Exercise



Of the animals used by the police department, 80% are dogs. The remainder are horses. Officer Daye is in charge of one of the animals used by the police department.

From the information given above, it can be validly concluded that, the animal in the charge of Officer Daye

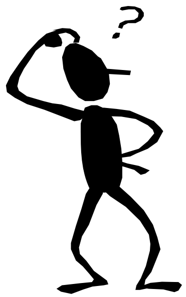
A) valid conclusion:

B) invalid conclusion:

C) invalid conclusion:

D) invalid conclusion:

Exercise



From the information given above, it can be validly concluded that, the animal in the charge of Officer Daye

Valid Conclusions

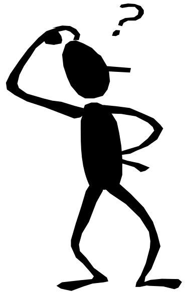
B1 is a dog, with a probability of .8

B2 is not a dog, with a probability of .2

B3 is not a horse, with a probability of .8

B4 is a horse, with a probability of .2

Exercise



From the information given above, it can be validly concluded that, the animal in the charge of Officer Daye

Invalid Conclusions
B5 may be a dog, but the probability cannot be determined
B6 may not be a dog, but the probability cannot be determined
B7 is not a dog, with a probability of .8
B8 is a dog, with a probability of .2
B9 may not be a horse, but the probability cannot be determined
B10 may be a horse, but the probability cannot be determined
B11 is a horse, with a probability of .8
B12 is not a horse, with a probability of .2

Using the Taxonomy

Table C:

C Premises Of all S, m/n are P. *a* is a P.

Valid Conclusions

C1 with an unknown prob., *a* is an S

C2 with an unknown prob., *a* is not an S

C3 with an unknown prob., *a* is not a non-S

C4 with an unknown prob., *a* is a non-S

Using the Taxonomy

Table C:

C Premises Of all S, m/n are P. *a* is an P.

Invalid Conclusions

C5 with a prob.of m/n, *a* is an S

C6 with a prob.of m/n, *a* is not an S

C7 with a prob.of m/n, *a* is a non-S

C8 with a prob.of m/n, *a* is not a non-S

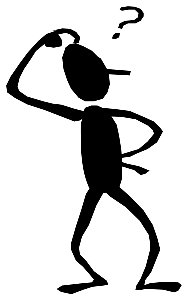
C9 with a prob. of 1 – m/n, *a* is an S

C10 with a prob. of 1 – m/n, *a* is not an S

C11 with a prob. of 1 – m/n, *a* is a non-S

C12 with a prob. of 1 – m/n, *a* is not a non-S

Exercise



Of all first-line supervisors in the department, 95% have taken the basic supervisory course. Rachel has taken the basic supervisory course.

From the information given above, it can be validly concluded that

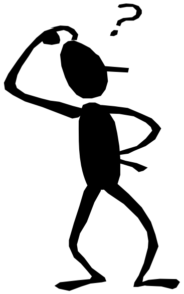
A) valid conclusion:

B) invalid conclusion:

C) invalid conclusion:

D) invalid conclusion:

Exercise



Valid Conclusions

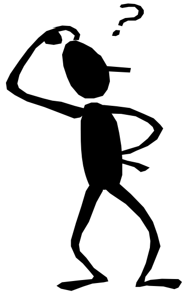
C1 Rachel may be a first-line supervisor, but the probability cannot be determined

C2 Rachel may not be a first-line supervisor, but the probability cannot be determined

C3 Rachel may not be an employee other than a first-line supervisor, but the probability cannot be determined

C4 Rachel may be a non-first-line supervisor, but the probability cannot be determined

Exercise



Invalid Conclusions

C5 with a probability of .95, Rachel is a first-line supervisor

C6 with a probability of .95, Rachel is not a first-line supervisor

C7 with a probability of .95, Rachel is an employee other than a first-line supervisor

C8 with a probability of .95, Rachel is not a non-first-line supervisor

C9 with a probability of .05, Rachel is a first-line supervisor

C10 with a probability of .05, Rachel is not a first-line supervisor

C11 with a probability of .05, Rachel is a non-first-line supervisor

C12 with a probability of .05, Rachel is not a non-first-line supervisor

Using the Taxonomy

Table F:

F Premises Of all S, m/n are not P. *a* is an S.

Valid Conclusions

F1 with a prob. of m/n, *a* is not a P

F2 with a prob. of 1 - m/n, *a* is a P

F3 with a prob. of m/n, *a* is a non-P

F4 with a prob. of 1 - m/n, *a* is not a non-P

Using the Taxonomy

Table F:

F Premises Of all S, m/n are not P. *a* is an S.

Invalid Conclusions

F5 with an unknown prob., *a* is not a P

F6 with an unknown prob., *a* is a P

F7 with a prob. of m/n, *a* is a P

F8 with a prob. of 1 – m/n, *a* is not a P

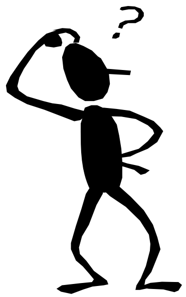
F9 with an unknown prob., *a* is a non-P

F10 with an unknown prob., *a* is not a non-P

F11 with a prob. of m/n, *a* is not a non-P

F12 with a prob. of 1 – m/n, *a* is a non-P

Exercise



Of the temporary employees, 35% are not college graduates. Instead, they are technical school graduates. Allen is one of the temporary employees.

From the information given above, it can be validly concluded that

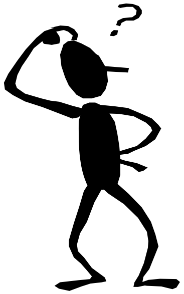
A) valid conclusion:

B) invalid conclusion:

C) invalid conclusion:

D) invalid conclusion:

Exercise



Valid Conclusions

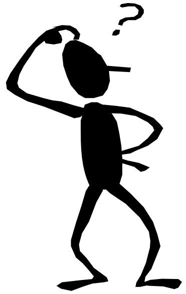
F1 with a probability of .35, Allen is not a college graduate

F2 with a probability of .65, Allen is a college graduate

F3 with a probability of .35, Allen is a technical school graduate

F4 with a probability of .65, Allen is not a technical school graduate

Exercise



Invalid Conclusions

F5 with an unknown probability, Allen is not a college graduate

F6 with an unknown probability, Allen is a college graduate

F7 with a probability of .35, Allen is a college graduate

F8 with a probability of .65, Allen is not a college graduate

F9 with an unknown probability, Allen is a technical school graduate

F10 with an unknown probability, Allen is not a technical school graduate

F11 with a probability of .35, Allen is not a technical school graduate

F12 with a probability of .65, Allen is a technical school graduate

Using the Taxonomy

Table G:

G Premises Of all S, m/n are not P. *a* is a P.

Valid Conclusions

G1 with an unknown prob., *a* is not an S

G2 with an unknown prob., *a* is an S

G3 with an unknown prob., *a* is a non-S

G4 with an unknown prob., *a* is not a non-S

Using the Taxonomy

Table G:

G Premises Of all S, m/n are not P. *a* is an P.

Invalid Conclusions

G5 with a prob.of m/n, *a* is not an S

G6 with a prob.of m/n, *a* is an S

G7 with a prob.of m/n, *a* is not a non-S

G8 with a prob.of m/n, *a* is a non-S

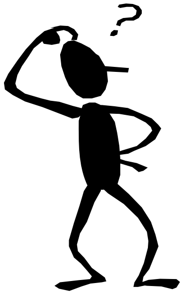
G9 with a prob. of 1 – m/n, *a* is not an S

G10 with a prob. of 1 – m/n, *a* is an S

G11 with a prob. of 1 – m/n, *a* is not a non-S

G12 with a prob. of 1 – m/n, *a* is a non-S

Exercise



Three-fourths of the crew aboard the cruise ship Amalie are not from Norway. Nora is aboard the Amalie, and she is from Norway.

From the information given above, it can be validly concluded that

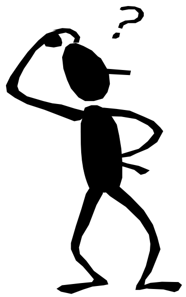
A) valid conclusion:

B) invalid conclusion:

C) invalid conclusion:

D) invalid conclusion:

Exercise



Valid Conclusions

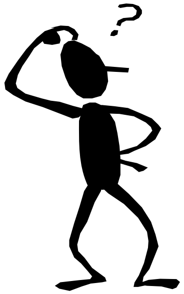
G1 Nora may not be a member of the crew, but the probability cannot be determined

G2 Nora may be a member of the crew, but the probability cannot be determined

G3 Nora may be a passenger, but the probability cannot be determined

G4 Nora may not be a passenger, but the probability cannot be determined

Exercise



Invalid Conclusions

G5 with a probability of .75, Nora is not a member of the crew

G6 with a probability of .75, Nora is a member of the crew

G7 with a probability of .75, Nora is not a passenger

G8 with a probability of .75, Nora is a passenger

G9 with a probability of .25, Nora is not a member of the crew

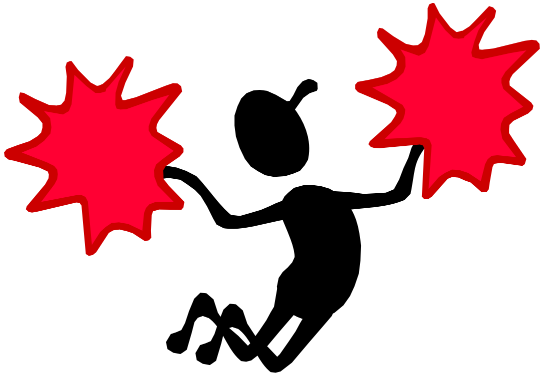
G10 with a probability of .25, Nora is a member of the crew

G11 with a probability of .25, Nora is not a passenger

G12 with a probability of .25, Nora is a passenger

Logic-Based Measurement

GOING LIVE!!



Writing a Test Question

A certain forensic laboratory performs many tests for law enforcement. Of the firearms received, 75% are .38 caliber. Also, 15% of the firearms are .22 caliber. Recently, officers brought firearm B1X3 to the forensic laboratory for testing.

From the information given above, it can be validly concluded that

A) valid conclusion:

B) invalid conclusion:

C) invalid conclusion:

D) invalid conclusion:

Writing a Test Question

Valid Conclusions

B1 with a probability of .75, B1X3 is a .38 caliber

B2 with a probability of .25, B1X3 is not a .38 caliber

B3 with a probability of .75, B1X3 is not a caliber other than .38

B4 with a probability of .25, B1X3 is a caliber other than .38

Writing a Test Question

Invalid Conclusions

B5 B1X3 may be a .38 caliber, but the probability cannot be determined

B6 B1X3 may not be a .38 caliber, but the probability cannot be determined

B7 with a probability of .75, B1X3 is not a .38 caliber

B8 with a probability of .25, B1X3 is a .38 caliber

B9 B1X3 may not be a caliber other than .38, but the probability cannot be determined

B10 B1X3 may be a caliber other than .38, but the probability cannot be determined

B11 with a probability of .75, B1X3 is a caliber other than .38

B12 with a probability of .25, B1X3 is not a caliber other than .38

Writing a Test Question

A new initiative is focused on stopping the illegal sale of the painkiller fentanyl. The initiative targets criminal groups responsible for the illegal distribution of fentanyl and other drugs. To date, the initiative has resulted in the arrest of 40 individuals. Of these individuals, 30 (75%) are members of the Balla gang. The initiative has also resulted in the discovery and dismantling of several narcotics labs. L.C. is a member of the Balla gang.

From the information given above, it can be validly concluded that

A) valid conclusion:

B) invalid conclusion:

C) invalid conclusion:

D) invalid conclusion:

Writing a Test Question

From the information given above, it can be validly concluded that, concerning the people arrested through the initiative against fentanyl,

Valid Conclusions

C1 L.C. may be one of the arrested individuals, but the probability cannot be determined

C2 L.C. may not be one of the arrested individuals, but the probability cannot be determined

C3 L.C. may not be someone other than one of the arrested individuals, but the probability cannot be determined

C4 L.C. may be someone other than one of the arrested individuals, but the probability cannot be determined

Writing a Test Question

From the information given above, it can be validly concluded that, concerning the people arrested through the initiative against fentanyl,

Invalid Conclusion

C5 with a probability of .75, L.C. is one of the arrested individuals

C6 with a probability of .75, L.C. is not one of the arrested individuals

C7 with a probability of .75, L.C. is someone other than one of the arrested individuals

C8 with a probability of .75, L.C. is not someone other than one of the arrested individuals

C9 with a probability of .25, L.C. is one of the arrested individuals

C10 with a probability of .25, L.C. is not one of the arrested individuals

C11 with a probability of .25, L.C. is someone other than one of the arrested individuals

C12 with a probability of .25, L.C. is not someone other than one of the arrested individuals

Examples of Inductive Reasoning

Preparation Manual for the ICE Special Agent Test Battery

http://www.ice.gov/careers/testprep/index.htm

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THANK YOU

Taxonomy For Reasoning Questions Using Logic-Based Measurement

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Taxonomy for Logic-Based Measurement

Introduction

This taxonomy should be used as a blueprint for both developing and documenting tests of job-related thinking skills. The thinking skills presented in the taxonomy are the basic forms of deductive and inductive reasoning. These forms of reasoning are the building blocks of complex forms of reasoning, such as decision-making.

The basic forms of deductive reasoning are divided into four parts for this taxonomy, and the basic forms of inductive reasoning are divided into two parts. Each part covers a different area of the domain of reasoning. Unlike other taxonomies, this taxonomy presents both correct and incorrect responses possible for each area of reasoning, enabling the test developer to be as sure of the "incorrectness" of incorrect responses as the "correctness" of correct responses.

In all parts of the taxonomy, tables are given that first show a certain type of premise or certain types of premises and that provide the valid and invalid conclusions for the premise or premises shown. Part A covers deductive reasoning from a single premise. The premise is a statement containing two sets. The conclusions in Part A are a single statement containing two sets. Part B covers deductive reasoning from two premises. Each premise is a statement that contains two sets. The two premises have one set in common. The conclusions are a single statement containing two of the three sets in the premises. Part C covers deductive reasoning with two statements that are connected. The emphasis in this part is on how the statements are connected instead of the sets that comprise the connected statements. Part D covers deductive reasoning with three connected statements. As in Part C, the emphasis in Part D is on how the statements are connected. Part E covers inductive reasoning from two premises. The initial premise is a statement containing two sets. The second premise is an individual member of a set. The conclusions in Part E are probabilistic conclusions about an individual member of a set. Part F covers inductive reasoning with two statements that are connected. The conclusions in Part F are probabilistic conclusions about a statement.

Taxonomy for Logic-Based Measurement

Part A: Deductive Reasoning with Two Sets: Tables A, E, I, and O

In Part A, four tables are given showing the valid and invalid conclusions based on the four basic types of two-set premises. Each premise is a single statement containing two sets, and each conclusion is a single statement containing two sets. The first set of the premise is denoted by "S" and the second set is denoted by "P."

Table A: "all are"
One Premise with Two Sets and the Quantifier All

A	Premise	All S are P.
A1	Valid Conclusion	No S are non-P.
A2		No non-P are S.
A3		Some P are S.
A4		All non-P are non-S.
A5	Invalid Conclusion	No S are P.
A6		Some S are not P.
A7		Some P are not S.
A8		All P are S.*
A9		All S are non-P.
A10		All P are non-S.
A11		No P are S.

*Illogical Bias

Table E: "no are "
One Premise with Two Sets and the Quantifier No

E	Premise	No S are P.
E1	Valid Conclusion	No P are S.
E2		All S are non-P.
E3		All P are non-S.
E4		Some P are not S.
E5	Invalid Conclusion	All S are P.
E6		All P are S.
E7		Some S are P.
E8		Some P are S.
E9		All non-S are P.
E10		All non-P are S.
E11		No non-P are non-S.*

*Illogical Bias

Taxonomy for Logic-Based Measurement

Table I: "some are"
One Premise with Two Sets and the Quantifier Some

I	Premise	Some S are P.
I1	Valid Conclusion	Some P are S.
I2		Some P are not non-S.
I3		Some S are not non-P.
I4	Invalid Conclusion	All S are P.
I5		No S are P.
I6		Some S are not P.*
I7		All P are S.
I8		No P are S.
I9		Some P are not S.
I10		Some non-P are non-S.*

*Illogical Bias

Table O: "some are not"
One Premise with Two Sets, the Quantifier Some

O	Premise	Some S are not P.
O1	Valid Conclusion	Some S are non-P.
O2		Some non-P are S.
O3	Invalid Conclusion	All S are P.
O4		No S are P.
O5		Some S are P.*
O6		Some P are not S.*
O7		No P are S.
O8		All P are S.

*Illogical Bias

Taxonomy for Logic-Based Measurement

Part B: Deductive Reasoning with Three Sets: Tables MA, ME, MI, and MO

In Part B, four tables are given showing the valid and invalid conclusions based on the four basic sets of conclusions for two-premise syllogisms. Each premise in a syllogism is a single statement containing two sets, and each conclusion is a single statement containing two sets. The two premises have one set in common, denoted by "M." The other two sets in the premises are denoted by "S" and by "P" as shown in the tables.

Table MA: Two Premises with Three Sets: S, M, and P

	Conclusions	
1	Valid Conclusion	All S are P.
2		No S are non-P.
3		No non-P are S.
4		Some P are S.
5		All non-P are non-S.
6	Invalid Conclusion	No S are P.
7		Some S are not P.
8		Some P are not S.
9		All P are S.
10		All S are non-P.
11		All P are non-S.
12		No P are S.

Name	Premises	Type	Logical Statement
1AA	Premise P	A	All M are P.
	Premise S	A	All S are M.

Table ME: Two Premises with Three Sets: S, M, and P

	Conclusions	
1	Valid Conclusion	No S are P.
2		No P are S.
3		All S are non-P.
4		All P are non-S.
5		Some P are not S.
6		Some S are not P.
7	Invalid Conclusion	All S are P.
8		All P are S.
9		Some S are P.
10		Some P are S.
11		All non-S are P.
12		All non-P are S.
13		No non-P are non-S.

Name	Premises	Type	Logical Statement
1EA	Premise P	E	No M are P.
	Premise S	A	All S are M.
2AE	Premise P	A	All P are M.
	Premise S	E	No S are M.
2EA	Premise P	E	No P are M.
	Premise S	A	All S are M.
4AE	Premise P	A	All P are M.
	Premise S	E	No M are S.

Taxonomy for Logic-Based Measurement

Table MI: Two Premises with Three Sets: S, M, and P

	Conclusions	
1	Valid Conclusion	Some S are P.
2		Some P are S.
3		Some P are not non-S.
4		Some S are not non-P.
5	Invalid Conclusion	All S are P.
6		No S are P.
7		Some S are not P.
8		All P are S.
9		No P are S.
10		Some P are not S.

Name	Premises	Type	Logical Statement
1AI	Premise P	A	All M are P.
	Premise S	I	Some S are M.
3AA	Premise P	A	All M are P.
	Premise S	A	All M are S.
3AI	Premise P	A	All M are P.
	Premise S	I	Some M are S.
3IA	Premise P	I	Some M are P.
	Premise S	A	All M are S.
4IA	Premise P	I	Some P are M.
	Premise S	A	All M are S.

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Table MO: Two Premises with Three Sets: S, M, and P

	Conclusions	
1	Valid Conclusion	Some S are not P.
2		Some S are non-P.
3		Some non-P are S.
4	Invalid Conclusion	All S are P.
5		No S are P.
6		Some S are P.
7		Some P are not S.
8		No P are S.
9		All P are S.

Name	Premises	Type	Logical Statement
1EI	Premise P	E	No M are P.
	Premise S	I	Some S are M.
2AO	Premise P	A	All P are M.
	Premise S	O	Some S are not M.
2EI	Premise P	E	No P are M.
	Premise S	I	Some S are M.
3EA	Premise P	E	No M are P.
	Premise S	A	All M are S.
3EI	Premise P	E	No M are P.
	Premise S	I	Some M are S.
3OA	Premise P	O	Some M are not P.
	Premise S	A	All M are S.
4EA	Premise P	E	No P are M.
	Premise S	A	All M are S.
4EI	Premise P	E	No P are M.
	Premise S	I	Some M are S.

Taxonomy for Logic-Based Measurement

Part C: Deductive Reasoning with Two Connected Statements: Tables S and T

In Part C, two tables are given showing the valid and invalid conclusions based on two basic types of connected statements. Each premise is a complex statement containing two statements, and each conclusion is complex statement containing two statements. The first statement of the premise is denoted by "p" and the second statement is denoted by "q."

The statements denoted by "p" and "q" can be the four basic two-set statements discussed in Parts A and B: All S are P, No S are P, Some S are P, and Some S are not P. If any of the four statements is used for "p" or "q," care must be taken in creating the negation of the statement. The following table shows the negation of the four basic statements.

Statement "p" (or "q")	**Negated statement "non-p" (or "non-q")**
All S are P	Some S are not P
No S are P	Some S are P
Some S are P	No S are P
Some S are not P	All S are P

Equivalencies of the Conditional Statement

The basic conditional statement has many equivalent statements. Some of these equivalent statements are merely different English phrasings of the same conditional statement (such as E2 below) and others are logically different from, but truth functionally equivalent to, the basic conditional statement (such as E5 below). These equivalencies may be used with valid and invalid response options.

	Statement	**Equivalence**
EQ1	if p then q	p only if q
EQ2	if p then q	q if p
EQ3	if p then q	not p unless q
EQ4	if p then q	not (both p and not-q)
EQ5	if p then q	either not-p or q

Table S: Two Statements Connected; p and q

S	Premise	if p then q
S1	Valid Conclusion	if p, then q
S2		if non-q, then non-p
S3	Invalid Conclusion	if p then non-q
S4		if non-p then q
S5		if non-p then non-q*
S6		if q then p*
S7		if q then non-p
S8		if non-q then p

*Illogical Bias

Table T: Two Statements Connected; p and q

T	Premise	p if and only if q
T1	Valid Conclusion	p if and only if q
T2		non-p if and only if non-q
T3		q if and only if p
T4		non-q if and only if non-p
T5		if p, then q
T6		if non-q, then non-p
T7		if q, then p
T8		if non-p, then non-q
T9	Invalid Conclusion	p if and only if non-q
T10		non-p if and only if q
T11		q if and only if non-p
T12		non-q if and only if p
T13		if p, then non-q
T14		if non-p, then q
T15		if q, then non-p
T16		if non-q, then p

Taxonomy for Logic-Based Measurement

Part D: Deductive Reasoning with Three Connected Statements: Table RS

In Part D, a table is given showing the valid and invalid conclusions for a syllogism based on two connected statements. Each premise is a complex statement containing two statements, and each conclusion is complex statement containing two statements. The two premises have one statement in common, denoted by "r." The other two statements in the premises are denoted by "p" and "q" as shown in the table.

Note: The equivalencies of the conditional statement apply here also.

	Statement	**Equivalence**
EQ1	if p then q	p only if q
EQ2	if p then q	q if p
EQ3	if p then q	not p unless q
EQ4	if p then q	not (both p and not-q)
EQ5	if p then q	either not-p or q

Table RS: Three Statements Connected; p, q, and r

	Premise	if r then q
	Premise	if p then r
RS1	Valid Conclusion	if p, then q
RS2		if non-q, then non-p
RS3	Invalid Conclusion	if p then non-q
RS4		if non-p then q
RS5		if non-p then non-q*
RS6		if q then p*
RS7		if q then non-p
RS8		if non-q then p

*Illogical Bias

Taxonomy for Logic-Based Measurement

Part E: Inductive Reasoning with Two Sets: Tables B, C, F and G

In Part E, four tables are given showing the valid and invalid conclusions based on two-set premises. The first set of the initial premise is denoted by "S," and the second set is denoted by "P."

Table B:
Two Premises with Two Sets and the Quantifier in the Initial Premise

B	Premises	Of all S, m/n are P. a is an S
B1	Valid Conclusion	with a probability of m/n, a is a P
B2		with a probability of 1 - m/n, a is not a P
B3		with a probability of m/n, a is not a non-P
B4		with a probability of 1 - m/n, a is non-P
B5	Invalid Conclusion	with an unknown probability, a is a P
B6		with an unknown probability, a is not a P
B7		with a probability of m/n, a is not a P
B8		with a probability of 1 - m/n, a is a P
B9		with an unknown probability, a is not a non-P
B10		with an unknown probability, a is a non-P
B11		with a probability of m/n, a is a non-P
B12		with a probability of 1 - m/n, a is not a non-P

Table C:
Two Premises with Two Sets and the Quantifier in the Initial Premise

C	Premises	Of all S, m/n are P. a is an P
C1	Valid Conclusion	with an unknown probability, a is an S
C2		with an unknown probability, a is not an S
C3		with an unknown probability, a is not a non-S
C4		with an unknown probability, a is a non-S
C5	Invalid Conclusion	with a probability of m/n, a is an S*
C6		with a probability of m/n, a is not an S
C7		with a probability of m/n, a is a non-S
C8		with a probability of m/n, a is not a non-S*
C9		with a probability of 1 - m/n, a is an S
C10		with a probability of 1 - m/n, a is not an S
C11		with a probability of 1 - m/n, a is a non-S
C12		with a probability of 1 - m/n, a is not a non-S

*Illogical Bias

Table F:
Two Premises with Two Sets and the Quantifier in the Initial Premise

F	Premises	Of all S, m/n are not P. a is an S
F1	Valid Conclusion	with a probability of m/n, a is not a P
F2		with a probability of 1 - m/n, a is a P
F3		with a probability of m/n, a is a non-P
F4		with a probability of 1 - m/n, a is not a non-P
F5	Invalid Conclusion	with an unknown probability, a is not a P
F6		with an unknown probability, a is a P
F7		with a probability of m/n, a is a P
F8		with a probability of 1 - m/n, a is not a P
F9		with an unknown probability, a is a non-P
F10		with an unknown probability, a is not a non-P
F11		with a probability of m/n, a is not a non-P
F12		with a probability of 1 - m/n, a is a non-P

Table G:
Two Premises with Two Sets and the Quantifier in the Initial Premise

G	Premises	Of all S, m/n are not P. a is an P
G1	Valid Conclusion	with an unknown probability, a is not an S
G2		with an unknown probability, a is an S
G3		with an unknown probability, a is a non-S
G4		with an unknown probability, a is not a non-S
G5	Invalid Conclusion	with a probability of m/n, a is not an S*
G6		with a probability of m/n, a is an S
G7		with a probability of m/n, a is not a non-S
G8		with a probability of m/n, a is a non-S*
G9		with a probability of 1 - m/n, a is not a S
G10		with a probability of 1 - m/n, a is an S
G11		with a probability of 1 - m/n, a is not a non-S
G12		with a probability of 1 - m/n, a is a non-S

*Illogical Bias

Taxonomy for Logic-Based Measurement

Part F: Inductive Reasoning with Two Connected Statements: Tables V and W

In Part F, two tables are given showing the valid and invalid conclusions based on connected statements. The first statement of the initial premise is denoted by "p," and the second statement is denoted by "q."

Table V:
Two Premises with Connected Statements and the Probability in the Initial Premise

V	Premises	if p, then q, with a probability of m/n. p
V1	Valid Conclusion	with a probability of m/n, q
V2		with a probability of 1 - m/n, not q
V3	Invalid Conclusion	with an unknown probability, q
V4		with an unknown probability, not q
V5		with a probability of m/n, not q
V6		with a probability of 1 - m/n, q

Table W:
Two Premises with Connected Statements and the Probability in the Initial Premise

W	Premises	if p, then q, with a probability of m/n. q
W1	Valid Conclusion	with an unknown probability, p
W2		with an unknown probability, not p
W3	Invalid Conclusion	with a probability of m/n, p*
W4		with a probability of m/n, not p
W5		with a probability of 1 - m/n, p
W6		with a probability of 1 - m/n, not p

*Illogical Bias