Inductive Reasoning with Connectives

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How is Inductive Reasoning Different from Deductive Reasoning?

Logicians have defined two types of reasoning: deductive and inductive.

- Inductive reasoning is reasoning with incomplete information and drawing a conclusion that may not be true, but which has some probability of being true.
- Deductive reasoning is reasoning with complete information to draw a certain or necessary conclusion.

How is Inductive Reasoning Different from Deductive Reasoning?

Historical note on Induction in psychometrics:

In early psychometric research, induction was defined as reasoning from the particular to general or discovering a rule.

- Thurstone, 1938, Primary Mental Abilities study:

  Induction was defined as that characteristic of tests which requires subjects "to find a rule or principle for each item in the test."

- This incomplete definition persisted into the late 20th century. In Carroll's 1993 compendium, Human Cognitive Abilities, inductive tasks were defined as those in which the subject is required "to inspect a set of materials and… "induce a rule governing the materials…"

- Typical questions used for measuring induction as thus defined are letter series and figure analogies.

For example: Letter series

Choose the answer that continues the following series:

b n c d n e f g n h l j k

How is Inductive Reasoning Different from Deductive Reasoning?

Inductive reasoning

- occurs in the absence of complete information
- leads to valid conclusions that are not **necessarily** true, but only have some probability of being true

Deductive reasoning

- occurs in the presence of complete information
- leads to valid conclusions that are **necessarily** true, if the evidence is true

How is Inductive Reasoning Different from Deductive Reasoning?

Inductive reasoning

- The evidence does not guarantee the truth of the conclusion, but it gives us a good reason to believe in the truth of the conclusion. The premises support the conclusion.

Deductive reasoning

- The truth of the evidence makes the truth of the conclusion certain.

How are Inductive Questions Different from Deductive Questions?

Example of deductive schemas

- *Premise:* Of all first-line supervisors in the agency, 95% have taken the basic supervisory course.

Conclusions:

- Almost no first-line supervisors in the agency have failed to take the basic supervisory course.
- At least some people who have taken the basic supervisory course are first-line supervisors in the agency.

How are Inductive Questions Different from Deductive Questions?

Example of inductive schemas

- *Premise:* Of all first-line supervisors in the agency, 95% have taken the basic supervisory course.
- Pat is one of the first-line supervisors.
- *Conclusion:*
- It is very likely that Pat has taken the basic supervisory course.

Criteria for Correct Induction

- The degree of probability claimed for the conclusion must be supported by the premises.
  - Example of conclusion not supported by the premises:

It is very unlikely that Pat has taken the basic supervisory course.

Criteria for Correct Induction

- The total available evidence must be used in forming a conclusion.
- If you obtain additional evidence, you may need to revise your conclusion.
  - Example of additional evidence:
  - "Supervisors in Pat's sector have attended the basic supervisory course in lower-than-average proportions, because of other urgent initiatives in the last year."
  - How would you revise the conclusion about Pat's attendance at supervisory training?

The Meaning of Probability

- Probability is
  - the likelihood that a conclusion is true, given certain evidence.
  - the *degree of confirmation* for the conclusion provided by the premises. This is the common interpretation of probability in inductive logic.

The Meaning of Probability

- The word probability can also be used for *relative frequency* -- how often an event occurs relative to other possible events.
- Example about first-level supervisors in the agency:
  - 95 out of 100 of these supervisors had taken the basic course in supervision. The relative frequency of 95/100 provided our best estimate of the probability that any one first-level supervisor had attended the training.

The Meaning of Probability

- Probabilities vary between 0 and 1.
  - A value of 1 represents absolute confirmation -- a condition that applies only to deductive conclusions.
  - A value of 0 represents absolute exclusion -- a condition that applies only to deductive conclusions.
  - For values between 0 and 1, numerical values may or may not be assigned.
    - Example: the word “probably” means “more likely than not” or “with a probability greater than .5.”
    - In test questions, we commonly use the expression “ there is a 95% chance” rather than “with a .95 probability.”

Expressions of Probability

• Probability greater than .7	greater than a 70% chance
• Probability equal to or greater than .7	at least a 70% chance no less than a 70% chance
• Probability equal to .7	exactly a 70% chance
• Probability equal to or less than .7	up to a 70% chance no greater than a 70% chance no more than a 70% chance
• Probability less than .7	less than a 70% chance
• Probably	greater than a 50% chance more likely than not

Inductive Reasoning with Connectives

- **Conditionals**
- The premise is a conditional in which the consequent is true only with a certain probability when the antecedent is true.
  - *If an agent applies for every vacancy in Sector X, he or she will have a 30% chance of being promoted in Sector X this year.*
- Another way of saying this: The consequent is true for only a proportion of the time when the antecedent is true. This means that part of the time the antecedent will be true and the consequent will be untrue.
- This is like the statistical concept of *conditional probability*.

Inductive Reasoning with Connectives

Example of Schema:

- If an agent applies for every vacancy in Sector X, he or she will have a 30% chance of being promoted in Sector X this year.

  If p, then q, with a probability of .3.

- Agent Jones applies for every vacancy in Sector X this year.

  p

- Therefore, Agent Jones has a 30% chance of being promoted in Sector X this year.

  q, with a probability of .3.

Inductive Reasoning with Connectives

- Unlike the deductive conditional, this form does not permit a definite conclusion to be drawn if the consequent is denied.
  - *If an agent applies for every vacancy in Sector X, he or she has a 30% chance of being promoted in Sector X this year.*
  - *Agent Smith was not promoted in Sector X this year.*
  - Conclusion*: Agent Smith may have applied for every vacancy in Sector X, but the probability is unknown.*

Inductive Reasoning with Connectives

Example of Schema:

- If an agent applies for every vacancy in Sector X, he or she will have a 30% chance of being promoted in Sector X this year.

  If p, then q, with a probability of .3.

- Agent was not promoted in Sector X this year.

  ~q

- Therefore, Agent Smith may have applied for every vacancy in Sector X this year, but the probability is unknown.

  p, with an unknown probability

Inductive Reasoning with Connectives

The inverse:

- If an agent applies for every vacancy in Sector X, he or she will have a 30% chance of being promoted in Sector X this year.

  If p, then q, with a probability of .3.

- Agent Jones did not apply for every vacancy in Sector X.

  ~p

- Therefore, Agent Jones may not be promoted in Sector X this year, but the probability is unknown.

  ~q, with an unknown probability

Inductive Reasoning with Connectives

The converse:

- If an agent applies for every vacancy in Sector X, he or she will have a 30% chance of being promoted in Sector X this year.

  If p, then q, with a probability of .3.

- Agent Smith was promoted in Sector X this year.

  q

- Therefore, Agent Smith may have applied for every vacancy in Sector X this year, but the probability is unknown.

  p, with an unknown probability

Using the Taxonomy

Table V:

V Premises: If p, then q, with a probability of m/n.

p

Valid Conclusions

V1 with a prob.of m/n, q

V2 with a prob.of 1 - m/n, not-q

Invalid Conclusions

V3 with an unknown probability, q

V4 with an unknown probability, not q

V5 with a prob. of m/n, not q

V6 with a prob. of 1 – m/n, q

Using the Taxonomy

Premises: If Ms. Greene's visa is one of the confiscated visas, the chances are 93% that her visa is genuine. Ms. Greene's visa is one of the confiscated visas.

- Valid Conclusion:

  V2 with a probability of .07, Ms. Greene's visa is fraudulent

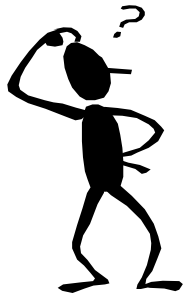
- Invalid Conclusions:

  V5 with a probability of .93, Ms. Greene's visa is not genuine

  V3 Ms. Greene's visa may be genuine, but the probability cannot be determined

  V6 with a probability of .07, Ms. Greene's visa is genuine

Exercise



The Border Patrol recently hired over 1000 new agents. These new agents will be stationed along the southwest border in either California or Texas. If a new agent completes training before April 1, there is a 35% chance that the agent will be stationed in Texas. Mr. Hamill is a new agent who will complete training before April 1.

From the information given above, it can be validly concluded that,

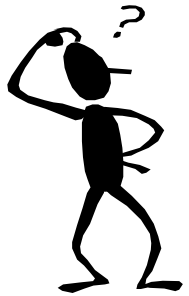
A) valid conclusion:

B) invalid conclusion:

C) invalid conclusion:

D) invalid conclusion:

Exercise

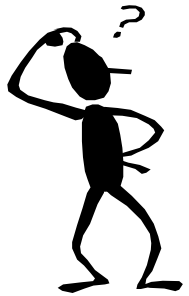


Valid Conclusions

V1 Mr. Hamill will be stationed in Texas, with a probability of .35

V2 Mr. Hamill will be stationed in California, with a probability of .65

Exercise



Invalid Conclusions

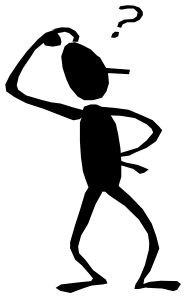
V3 Mr. Hamill may be stationed in Texas, but the probability cannot be determined

V4 Mr. Hamill may not be stationed in Texas, but the probability cannot be determined

V5 Mr. Hamill will not be stationed in Texas, with a probability of .35

V6 Mr. Hamill will be stationed in Texas, with a probability of .65

Exercise



The town runs a program that enables firefighters to live in moderate income housing more easily than those who are not firefighters. In fact, if a firefighter applies for moderate income housing, that individual has a 55% chance of attaining the housing, whereas a non-firefighter has only a 30% chance of attaining the housing. Ms. Justus is a firefighter in the town who has applied for moderate income housing.

From the information given above, it can be validly concluded that,

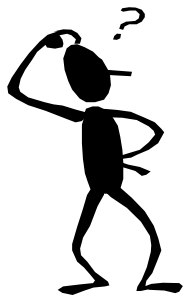
A) valid conclusion:

B) invalid conclusion:

C) invalid conclusion:

D) invalid conclusion:

Exercise

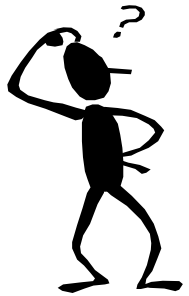


Valid Conclusions

V1 Ms. Justus will attain the moderate income housing, with a probability of .55

V2 Ms. Justus will not attain the moderate income housing, with a probability of .45

Exercise



Invalid Conclusions

V3 Ms. Justus may attain the moderate income housing, but the probability cannot be determined

V4 Ms. Justus may not attain the moderate income housing, but the probability cannot be determined

V5 Ms. Justus will not attain the moderate income housing, with a probability of .55

V6 Ms. Justus will attain the moderate income housing, with a probability of .45

Using the Taxonomy

Table W:

W Premises: If p, then q, with a probability of m/n.

q

Valid Conclusions

W1 with an unknown probability, p

W2 with an unknown probability, not p

Invalid Conclusions

W3 with a prob. of m/n, p

W4 with a prob. of m/n, not p

W5 with a prob. of 1 – m/n, p

W6 with a prob. of 1 – m/n, not p

Using the Taxonomy

Premises: If Ms. Greene's visa is one of the confiscated visas, the chances are 93% that her visa is genuine. Ms. Greene's visa is genuine.

- Valid Conclusion:

  W1 Ms. Greene's visa may be one of the confiscated visas, but the probability cannot be determined

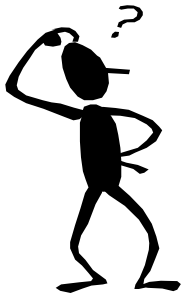
- Invalid Conclusions:

  W3 with a probability of .93, Ms. Greene's visa is one of the confiscated visas

  W4 with a probability of .93, Ms. Greene's visa is not one of the confiscated visas

  W5 with a probability of .07, Ms. Greene's visa is one of the confiscated visas

Exercise



There are several reasons why customers of Acme Inc. file complaints. For example, sometimes customers calling into the refund call center are unsatisfied with the call and file a complaint. In fact, if a customer calls the refund call center, the chances are 8% that the customer will file a complaint. Ms. Engle is a Acme Inc. customer who filed a complaint.

From the information given above, it can be validly concluded that,

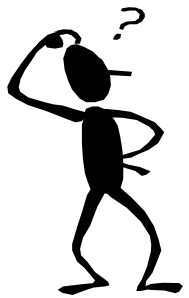
A) valid conclusion:

B) invalid conclusion:

C) invalid conclusion:

D) invalid conclusion:

Exercise

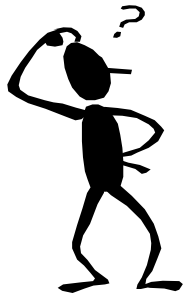


Valid Conclusions

W1 Ms. Engle may have called the refund call center, but the probability cannot be determined

W2 Ms. Engle may not have called the refund call center, but the probability cannot be determined

Exercise



Invalid Conclusions

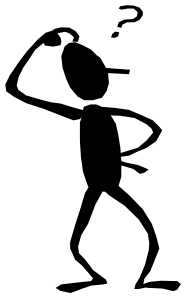
W3 with a probability of .08, Ms. Engle called the refund call center

W4 with a probability of .08, Ms. Engle did not call the refund call center

W5 with a probability of .92, Ms. Engle called the refund call center

W6 with a probability of .92, Ms. Engle did not call the refund call center

Exercise



The town charges a $25 parking fine for parking in a permit zone without the proper permit and for parking at an expired parking meter. Based on yearly statistics, if you park in a permit zone without the proper permit, the chances of receiving a $25 parking fine are 65%. Mr. Vasil received a $25 parking fine from the town.

From the information given above, it can be validly concluded that,

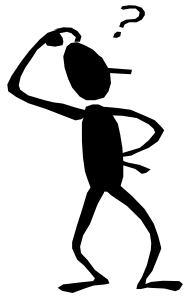
A) valid conclusion:

B) invalid conclusion:

C) invalid conclusion:

D) invalid conclusion:

Exercise

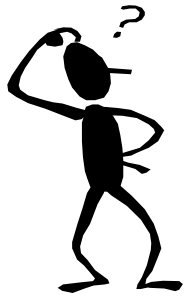


Valid Conclusions

W1 Mr. Vasil may have parked in a permit zone without the proper permit, but the probability cannot be determined

W2 Mr. Vasil may not have parked in a permit zone without the proper permit, but the probability cannot be determined

Exercise



Invalid Conclusions

W3 with a probability of .65, Mr. Vasil parked in a permit zone without the proper permit

W4 with a probability of .65, Mr. Vasil did not park in a permit zone without the proper permit

W5 with a probability of .35, Mr. Vasil parked in a permit zone without the proper permit

W6 with a probability of .35, Mr. Vasil did not park in a permit zone without the proper permit

Inductive Reasoning with Connectives

- **Biconditionals**
- We can define a biconditional premise that has two conditional probabilities associated with it. It would retain the symmetry associated with the deductive biconditional. Such a biconditional would look like this:
  - Of all validation study participants who obtained a passing score on the test, 50% had superior scores in IOBTC.
  - Of all validation study participants who had superior scores in IOBTC, 90% had a passing score on the test.

Inductive Reasoning with Connectives

If a validation study participant obtained a passing score on the test, there is a 50% chance that the participant had a superior score in IOBTC.

If p, then q, with a probability of .5.

If a validation study participant had a superior score in IOBTC, there is a 90% chance that the participant had a passing score on the test.

If q, then p, with a probability of .9.

What are the valid conclusions from these facts about the study participants?

A.B. obtained a passing score on the test.

C.D. had a superior score at IOBTC.

E.F. did not obtain a passing score in the test.

G.H. did not have a superior score at IOBTC.

Examples of LBM Tests

Preparation Manual for the ICE Special Agent Test Battery

http://www.ice.gov/careers/testprep/index.htm

U.S. Customs and Border Protection jobs

http://www.cbp.gov/xp/cgov/careers/study_guides/

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THANK YOU