

Using Logic-Based Testing to Develop Powerful Measures of Reasoning Ability - Workshop -

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Getting Started in LBM

Item writers must spend time becoming familiar with basic principles of logic.

- This workshop will begin the process of familiarization.

After becoming familiar with logic, the next step is to create or adapt a taxonomy of logical formulas.

- A taxonomy defines the content domain of the reasoning construct, both for the job and for the selection test
- A taxonomy is provided in this workshop



LBM Question

Usually an officer cannot search an individual without a warrant. However, there are some exceptions. For example, if the safety of an officer is involved, the officer may search an individual without a warrant.

From the information given above, it can be validly concluded that

- A) an officer may search an individual without a warrant if the safety of the officer is not involved
- B) if an officer may not search an individual without a warrant, then the safety of the officer is not involved
- C) if the safety of an officer is involved, the officer may not search an individual without a warrant
- D) an officer may search an individual without a warrant only if the safety of the officer is involved
- E) if the safety of an officer is not involved, then the officer may not search an individual without a warrant



Connectives

- Conditionals are part of the Logic of Connectives
- We will talk about:
 - Parts of Connective Statements
 - Types of Connective Statements
 - Valid Conclusions
 - Invalid Conclusions



Connective Statements

- Two types of parts:
 - 1) simple statements
 - 2) connectives, such as *if...then*

| Con- nective | Statement | Con- nective | Statement |
|-----------------|-----------------------------------|-----------------|-------------------------------------|
| If | a person is an employee of DHS | then | the person is a Federal employee |



Connective Statements

- Any one connective statement must have two simple statements and one connective.
- However, any one connective statement can have more than two simple statements and more than one connective: the compound conditional.

Example: embedded connective

| Con- nective | Statement | Con- nective | Statement |
|-----------------|---|-----------------|----------------------------------|
| If | a person is an employee of DHS or a person is an employee of DOJ | then | the person is a Federal employee |
| | | | |
| | a person is an employee of the DHS | or* | a person is an employee of DOJ |

*embedded connective: the 'or' is embedded within the main conditional 'if ... then'



Simple and Complete Statements

- Connective statements are compound sentences.
- The statements that make up the components of the compound sentences are of the form *A is B*.
- They are simple statements, but they are complete statements.
- For example, “John” is not a simple statement. “if a person is an officer, then John” is not a connective statement.



Simple Statements

- Example: If a person is an employee of DHS or of DOJ, then the person is a Federal employee.
- Connectives: if ... then, or
- Statements
 - a person is an employee of DHS
 - a person is an employee of DOJ
 - a person is a Federal employee

Exercise



Underline the simple statements:

- There is a record of a deduction on your biweekly earnings statement if you contribute to the Combined Federal Campaign through payroll deduction.
- You can take the advanced supervisory course only if you have taken the basic supervisory course.
- A person is European if the person is German.
- If an international flight arrives, CBP Officers process the arriving passengers.
- You can stay in the condo for free if and only if you attend the sales seminar.
- If a person is a CBP Officer, the person works for DHS.



Connectives

- if ... then (sometimes 'then' is tacit)
- only if
- when
- both ... and
- either ... or
- neither ... nor



Representation

- Simple statements are represented by lower case letters, such as p , q , r .
 - p = a person is an employee of DHS
 - q = a person is an employee of DOJ
 - r = a person is a Federal employee



Roadmap

- Conditional
- Biconditional
- Extended conditional



Conditional

- If A is B, then C is D.
- If p, then q.
- $p \supset q$.



Two Logical Parts

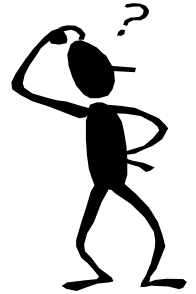
- The conditional statement has two logical parts, other than connectives;
 - 1) a condition
 - 2) a result of the condition being true
- Example: if the car is out of gas, the car is unable to run.



Conditional

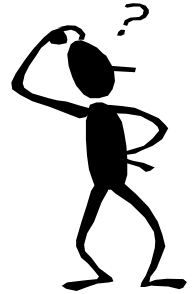
- Condition = antecedent, Result = consequent
 - If the car is out of gas, then the car will not run
 - antecedent = car is out of gas
 - consequent = the car will not run
- The conditional statement says that if the antecedent is true, then the consequent must also be true.
- If the student is eligible for this class, then he/she has completed the prerequisites.

Conditional



- The budget will be approved if the department requests a smaller spending level than last year.
- The deal will fall through if Lisa cannot attend the meeting.
- If a person is hired as a Border Patrol agent, the person attends training at FLETC.
- The neighborhood streets are dark during the day if there is a total eclipse of the sun.

Conditional



- The computer was purchased by David only if the computer lacks a floppy drive.
- Rachel is eligible only if she has signed a waiver.



Conditional

- ‘if’ is NOT equal to ‘only if’
- The bank is not open if today is a holiday.
(true: if p, then q)
- The bank is not open only if today is a holiday. (not true: if q, then p)
 - p = today is a holiday
 - q = the bank is not open



Manipulating Parts of a Connective Statement

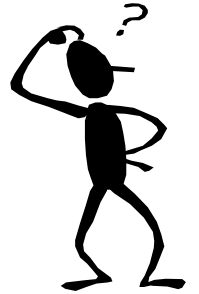
- Negating Simple Statements
- Exchanging Simple Statements
- Changing the Connectives



Negating Simple Statements

- *To negate a simple statement* is to reverse the quality of the simple statement.
- For example:
 - John is mad.
 - John is not mad.
 - Congress will adjourn before passing the legislation.
 - Congress will not adjourn before passing the legislation.

Negating Simple Statements



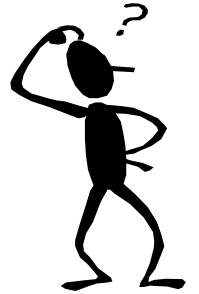
Exercise:

If the city is on terrorist alert, the building is closed to visitors.

Negated antecedent:

Negated consequent:

Negating Simple Statements



Exercise:

A non-immigrant alien has violated his or her non-immigrant status if the alien accepts unauthorized employment.

Negated antecedent:

Negated consequent:



Negating Simple Statements

- Logic note: When both the antecedent and the consequent are negated, the new statement is called the *inverse* of the original statement.
- Premise
 - If a non-immigrant alien enters the U.S. illegally, the alien is an “undocumented alien.”
 - If a non-immigrant alien enters the U.S. legally, the alien is not an “undocumented alien.” [Inverse]
 - The inverse of the conditional statement is an Illogical Bias



Exchanging Simple Statements

- *To exchange simple statements* is to make the antecedent become the consequent and make the consequent become the antecedent.
- **Logic note: the new statement is called the *converse* of the original statement.**
- **The converse of the conditional statement is an Illogical Bias**



Exchanging Simple Statements

- **Original Conditional Statement**

- **Statement:** An employer is permitted to hire an applicant only if the employer is able to verify that the applicant's employment documentation establishes that the applicant is authorized to work in the United States.
- **Antecedent:** An employer is permitted to hire an applicant
- **Consequent:** the employer is able to verify that the applicant's employment documentation establishes that the applicant is authorized to work in the United States
- **Symbols:** p only if q



Exchanging Simple Statements

- **Statement with Exchanged Simple Statements**
 - **Statement:** An employer is able to verify that an applicant's employment documentation establishes that the applicant is authorized to work in the United States only if the employer is permitted to hire the applicant .
 - **Antecedent:** an employer is able to verify that an applicant's employment documentation establishes that the applicant is authorized to work in the United States
 - **Consequent:** an employer is permitted to hire an applicant
 - **Symbols:** q only if p

Exchanging Simple Statements

EXERCISE:

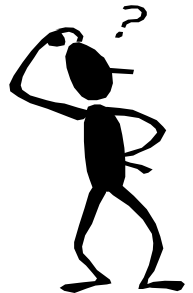
Statement: If the contract is valid, then the contract is notarized.

Antecedent: the contract is valid

Consequent: the contract is notarized

Logical Statement: if p , then q

Statement with terms exchanged:



Exchanging Simple Statements

EXERCISE:

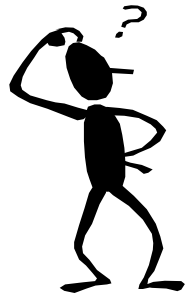
Statement: if an applicant has veterans' preference, the applicant has permanent reinstatement eligibility

Antecedent: an applicant has veterans' preference

Consequent: an applicant has permanent reinstatement eligibility

Logical Statement: if p , then q

Statement with terms exchanged:





Changing the Connectives

- Basic connectives: if/then, only if, if, and, or
- Equivalent connectives to if/then:
 - When/then, After/then
- *To change the connective* is to replace a given connective with another connective.



Changing the Connectives

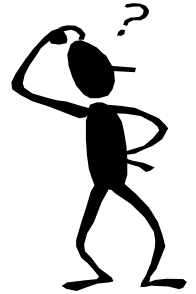
- If Smith's license is invalid, then he may not drive the company van.
 - If p , then q
- Smith's license is invalid only if he may not drive the company van.
 - p only if q (valid)
- Smith's license is invalid if he may not drive the company van.
 - p if q (invalid)

Exercise



- If an applicant is eligible to become a Federal law enforcement officer, then the applicant has no convictions of domestic violence. (if p , then q)
 - p only if q (valid)
 - p if q (invalid)

Putting It All Together



Premise: If a child is born in the U.S. while under U.S. jurisdiction, the child is a U.S. citizen at birth. (if p , then q)

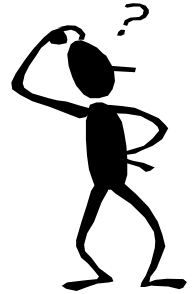
Write a statement that represents:

if non- p , then non- q

if q , then non- p

if non- q , then non- p

Putting It All Together



Premise: The operation will succeed only if the extraction team does not get caught. (p only if q)

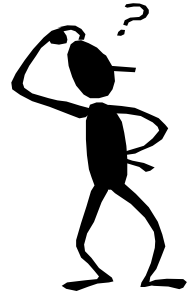
Write a statement that represents:

if p, then q

non-p only if non-q

if non-q, then non-p

Putting It All Together



Premise: A naturalized U.S. citizen loses U.S. citizenship if he or she expatriates.

Write a statement that represents:

if p , then q

non- p if non- q

if q , then p

LBM Question

Usually an officer cannot search an individual without a warrant. However, there are some exceptions. For example, if the safety of an officer is involved, the officer may search an individual without a warrant. (if p, then q)

From the information given above, it can be validly concluded that

- A) an officer may search an individual without a warrant if the safety of the officer is not involved (q if non-p)
- B) if an officer may not search an individual without a warrant, then the safety of the officer is not involved (if non-q, then non-p)
- C) if the safety of an officer is involved, the officer may not search an individual without a warrant (if p, then non-q)
- D) an officer may search an individual without a warrant only if the safety of the officer is involved (q only if p)
- E) if the safety of an officer is not involved, then the officer may



Using the Taxonomy

Table S:

S Premise If p , then q .

Valid Conclusions

S1 if p , then q .

S2 if non- q , then non- p .

Invalid Conclusions

S3 if p , then non- q

S4 if non- p , then q

S5 if non- p , then non- q

S6 if q , then p

S7 if q , then non- p

S8 if non- q , then p



Using the Taxonomy

Equivalencies

- Think of 'p' as the antecedent and 'q' as the 'consequent'
- EQ1, antecedent only if consequent
- EQ2, consequent if antecedent
- EQ3, not the antecedent unless the consequent
- EQ4, not (both the antecedent and the negated consequent)
- EQ5, either not the antecedent or else the consequent



Using the Taxonomy

S1 if p, then q

Equivalencies

S1, EQ1 p only if q

S1, EQ2 q if p

S1, EQ3 not p unless q

S1, EQ4 not (both p and non-q)

S1, EQ5 either non-p or q



Using the Taxonomy

S2 if non-q, then non-p

Equivalencies

S2, EQ1 non-q only if non-p

S2, EQ2 non-p if non-q

S2, EQ3 not non-q unless non-p

S2, EQ4 not (both non-q and p)

S2, EQ5 either q or non-p



Using the Taxonomy

S3 if p, then non-q

Equivalencies

S3, EQ1 p only if non-q

S3, EQ2 non-q if p

S3, EQ3 not p unless non-q

S3, EQ4 not (both p and q)

S3, EQ5 either non-p or non-q



Using the Taxonomy

- Building an LBM question with valid and invalid conclusions
- Steps
 - 1 Choose a statement for the premise
 - 2 Parse the statement logically
 - 3 Go to the table in the taxonomy that serves your premise
 - 4 Choose one valid conclusion
 - 5 Choose invalid conclusions



Using the Taxonomy

- Premise: If a person is a CBP Officer, then the person is an employee of the U.S. Government.
if p, then q
- Valid Conclusion:
S2 If a person is not an employee of the U.S. Government, then the person is not a CBP Officer.
- Invalid Conclusions:
S6 If a person is an employee of the U.S. Government, the person is a CBP Officer.
S6, EQ1 A person is an employee of the U.S. Government only if the person is a CBP Officer.
S8 If a person is not an employee of the U.S. Government, then the person is a CBP Officer.



Using the Taxonomy

- Premise: Bill and Shirley are workers at the same office. At this office, if a worker leaves the vault open, the worker will be dismissed. Bill left the vault open.
if p, then q; and p
- Valid Conclusion:
S1 Bill will be dismissed.
- Invalid Conclusions:
S3 Bill will not be dismissed.
S6, EQ1 Shirley will be dismissed only if she leaves the vault open.
S8, EQ2 Shirley left the vault open if she will not be dismissed.

Exercise



Premise: If a person has been convicted of murder, that person is a felon.

From the information given above, it can be validly concluded that

- A) valid conclusion:
- B) invalid conclusion:
- C) invalid conclusion:
- D) invalid conclusion:

Exercise




Premise: If a person is hired as a Border Patrol Agent trainee, the person will attend training at FLETC. Sherry has applied to become a Border Patrol Agent trainee.

From the information given above, it can be validly concluded that

- A) valid conclusion:
- B) invalid conclusion:
- C) invalid conclusion:
- D) invalid conclusion:

Roadmap

- Conditional 
- Biconditional
- Extended conditional



Biconditional

- If A is B, then C is D; and if C is D, then A is B
- If p then q; and if q then p
- p if and only if q
- $p \equiv q$
- The Secretary of the DHS is the director of your agency if and only if you are an employee of DHS.



Biconditional

- p if and only if q
 - p if q = if q , then p
 - p only if q = if p , then q
- if q , then p ; if p , then q



Biconditional

p if and only if q

- Valid (T5 - T8)
 - if p, then q if $\sim q$, then $\sim p$
 - if q, then p if $\sim p$, then $\sim q$
- Invalid (T13 - T16)
 - if p, then $\sim q$ if $\sim q$, then p
 - if q, then $\sim p$ if $\sim p$, then q



Example

Rhett and Abby both received a special offer to receive the free use of a condo in Florida. However, there was a catch. They were told that they can stay in the condo for free if and only if they attend the sales seminar. Rhett attended the sales seminar, but Abby did not.

$p \text{ iff } q$; and q (Rhett) and $\sim q$ (Abby)

- Valid Conclusion:

T7 Rhett can stay in the condo for free.

- Invalid Conclusions:

T16 Abby can stay in the condo for free.

T15 Rhett cannot stay in the condo for free.





Example

The local water utility chairman has been accused of providing false testimony. Although the utility's executive board wishes the chairman to resign because of the accusations, thus far the chairman has refused. In fact, the chairman has vowed to resign if and only if there is an actual conviction.

From the information given above, it can be validly concluded that, assuming that the chairman's vow is adhered to,

- A) the chairman has not been convicted if and only if the chairman has resigned - T12
- B) if the chairman has resigned, then there is an actual conviction – T5
- C) the chairman has been convicted if and only if the chairman has not resigned - T11

Roadmap

- Conditional 
- Biconditional 
- Extended conditional



Extended Conditional

If r, then q

if p, then r

therefore, if p, then q

$(r \supset q) \wedge (p \supset r); \wedge p; \therefore q.$



Extended Conditional

If additional staff are assigned, special funding will be needed. If it is a holiday weekend, additional staff are assigned.

Therefore, if it is a holiday weekend, special funding will be needed.

If \underline{r} , then q
if p , then \underline{r}
therefore, if p , then q



Inferences

If r , then q
if p , then r

Valid

therefore, if p , then q

therefore, if $\sim q$, then $\sim p$

Illogical Biases

therefore, if $\sim p$, then $\sim q$ inverse

therefore, if q , then p converse





Example

Impressions made by the ridges on the ends of the fingers and thumbs are useful means of identification. If finger patterns from fingerprints are not decipherable, then they cannot be classified by general shape and contour or by pattern type. If they cannot be classified by these characteristics, then it is impossible to identify the person to whom the fingerprints belong.

From the information given above, it can be validly concluded that

- A) if it is impossible to identify the person to whom fingerprints belong, then the fingerprints are not decipherable – RS6
- B) if finger patterns from fingerprints are not decipherable, then it is impossible to identify the person to whom the fingerprints belong – RS1
- C) if it is possible to identify the person to whom fingerprints belong, then the fingerprints cannot be classified by general shape and contour or pattern type - S8

Roadmap

- Conditional 
- Biconditional 
- Extended conditional 