Taxonomy For Reasoning Questions Using Logic-Based Measurement

Homeland Security

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Taxonomy for Logic-Based Measurement

Introduction

This taxonomy should be used as a blueprint for both developing and documenting tests of job-related thinking skills. The thinking skills presented in the taxonomy are the basic forms of deductive and inductive reasoning. These forms of reasoning are the building blocks of complex forms of reasoning, such as decision-making.

The basic forms of deductive reasoning are divided into four parts for this taxonomy, and the basic forms of inductive reasoning are divided into two parts. Each part covers a different area of the domain of reasoning. Unlike other taxonomies, this taxonomy presents both correct and incorrect responses possible for each area of reasoning, enabling the test developer to be as sure of the "incorrectness" of incorrect responses as the "correctness" of correct responses.

In all parts of the taxonomy, tables are given that first show a certain type of premise or certain types of premises and that provide the valid and invalid conclusions for the premise or premises shown. Part A covers deductive reasoning from a single premise. The premise is a statement containing two sets. The conclusions in Part A are a single statement containing two sets. Part B covers deductive reasoning from two premises. Each premise is a statement that contains two sets. The two premises have one set in common. The conclusions are a single statement containing two of the three sets in the premises. Part C covers deductive reasoning with two statements that are connected. The emphasis in this part is on how the statements are connected instead of the sets that comprise the connected statements. Part D covers deductive reasoning with three connected statements. As in Part C, the emphasis in Part D is on how the statements are connected. Part E covers inductive reasoning from two premises. The initial premise is a statement containing two sets. The second premise is an individual member of a set. The conclusions in Part E are probabilistic conclusions about an individual member of a set. Part F covers inductive reasoning with two statements that are connected. The conclusions in Part F are probabilistic conclusions about a statement.

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Part A: Deductive Reasoning with Two Sets: Tables A, E, I, and O

In Part A, four tables are given showing the valid and invalid conclusions based on the four basic types of two-set premises. Each premise is a single statement containing two sets, and each conclusion is a single statement containing two sets. The first set of the premise is denoted by "S" and the second set is denoted by "P."

Table A: "all are"
One Premise with Two Sets and the Quantifier All

A	Premise	All S are P.
A1	Valid Conclusion	No S are non-P.
A2		No non-P are S.
A3		Some P are S.
A4		All non-P are non-S.
A5	Invalid Conclusion	No S are P.
A6		Some S are not P.
A7		Some P are not S.
A8		All P are S.*
A9		All S are non-P.
A10		All P are non-S.
A11		No P are S.

*Illogical Bias

Table E: "no are "
One Premise with Two Sets and the Quantifier No

E	Premise	No S are P.
E1	Valid Conclusion	No P are S.
E2		All S are non-P.
E3		All P are non-S.
E4		Some P are not S.
E5	Invalid Conclusion	All S are P.
E6		All P are S.
E7		Some S are P.
E8		Some P are S.
E9		All non-S are P.
E10		All non-P are S.
E11		No non-P are non-S.*

*Illogical Bias

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Table I: "some are"
One Premise with Two Sets and the Quantifier Some

I	Premise	Some S are P.
I1	Valid Conclusion	Some P are S.
I2		Some P are not non-S.
I3		Some S are not non-P.
I4	Invalid Conclusion	All S are P.
I5		No S are P.
I6		Some S are not P.*
I7		All P are S.
I8		No P are S.
I9		Some P are not S.
I10		Some non-P are non-S.*

*Illogical Bias

Table O: "some are not"
One Premise with Two Sets, the Quantifier Some

O	Premise	Some S are not P.
O1	Valid Conclusion	Some S are non-P.
O2		Some non-P are S.
O3	Invalid Conclusion	All S are P.
O4		No S are P.
O5		Some S are P.*
O6		Some P are not S.*
O7		No P are S.
O8		All P are S.

*Illogical Bias

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Part B: Deductive Reasoning with Three Sets: Tables MA, ME, MI, and MO

In Part B, four tables are given showing the valid and invalid conclusions based on the four basic sets of conclusions for two-premise syllogisms. Each premise in a syllogism is a single statement containing two sets, and each conclusion is a single statement containing two sets. The two premises have one set in common, denoted by "M." The other two sets in the premises are denoted by "S" and by "P" as shown in the tables.

Table MA: Two Premises with Three Sets: S, M, and P

	Conclusions	
1	Valid Conclusion	All S are P.
2		No S are non-P.
3		No non-P are S.
4		Some P are S.
5		All non-P are non-S.
6	Invalid Conclusion	No S are P.
7		Some S are not P.
8		Some P are not S.
9		All P are S.
10		All S are non-P.
11		All P are non-S.
12		No P are S.

Name	Premises	Type	Logical Statement
1AA	Premise P	A	All M are P.
	Premise S	A	All S are M.

Table ME: Two Premises with Three Sets: S, M, and P

	Conclusions	
1	Valid Conclusion	No S are P.
2		No P are S.
3		All S are non-P.
4		All P are non-S.
5		Some P are not S.
6		Some S are not P.
7	Invalid Conclusion	All S are P.
8		All P are S.
9		Some S are P.
10		Some P are S.
11		All non-S are P.
12		All non-P are S.
13		No non-P are non-S.

Name	Premises	Type	Logical Statement
1EA	Premise P	E	No M are P.
	Premise S	A	All S are M.
2AE	Premise P	A	All P are M.
	Premise S	E	No S are M.
2EA	Premise P	E	No P are M.
	Premise S	A	All S are M.
4AE	Premise P	A	All P are M.
	Premise S	E	No M are S.

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Table MI: Two Premises with Three Sets: S, M, and P

	Conclusions	
1	Valid Conclusion	Some S are P.
2		Some P are S.
3		Some P are not non-S.
4		Some S are not non-P.
5	Invalid Conclusion	All S are P.
6		No S are P.
7		Some S are not P.
8		All P are S.
9		No P are S.
10		Some P are not S.

Name	Premises	Type	Logical Statement
1AI	Premise P	A	All M are P.
	Premise S	I	Some S are M.
3AA	Premise P	A	All M are P.
	Premise S	A	All M are S.
3AI	Premise P	A	All M are P.
	Premise S	I	Some M are S.
3IA	Premise P	I	Some M are P.
	Premise S	A	All M are S.
4IA	Premise P	I	Some P are M.
	Premise S	A	All M are S.

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Table MO: Two Premises with Three Sets: S, M, and P

	Conclusions	
1	Valid Conclusion	Some S are not P.
2		Some S are non-P.
3		Some non-P are S.
4	Invalid Conclusion	All S are P.
5		No S are P.
6		Some S are P.
7		Some P are not S.
8		No P are S.
9		All P are S.

Name	Premises	Type	Logical Statement
1EI	Premise P	E	No M are P.
	Premise S	I	Some S are M.
2AO	Premise P	A	All P are M.
	Premise S	O	Some S are not M.
2EI	Premise P	E	No P are M.
	Premise S	I	Some S are M.
3EA	Premise P	E	No M are P.
	Premise S	A	All M are S.
3EI	Premise P	E	No M are P.
	Premise S	I	Some M are S.
3OA	Premise P	O	Some M are not P.
	Premise S	A	All M are S.
4EA	Premise P	E	No P are M.
	Premise S	A	All M are S.
4EI	Premise P	E	No P are M.
	Premise S	I	Some M are S.

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Part C: Deductive Reasoning with Two Connected Statements: Tables S and T

In Part C, two tables are given showing the valid and invalid conclusions based on two basic types of connected statements. Each premise is a complex statement containing two statements, and each conclusion is complex statement containing two statements. The first statement of the premise is denoted by "p" and the second statement is denoted by "q."

The statements denoted by "p" and "q" can be the four basic two-set statements discussed in Parts A and B: All S are P, No S are P, Some S are P, and Some S are not P. If any of the four statements is used for "p" or "q," care must be taken in creating the negation of the statement. The following table shows the negation of the four basic statements.

Statement "p" (or "q")	**Negated statement "non-p" (or "non-q")**
All S are P	Some S are not P
No S are P	Some S are P
Some S are P	No S are P
Some S are not P	All S are P

Equivalencies of the Conditional Statement

The basic conditional statement has many equivalent statements. Some of these equivalent statements are merely different English phrasings of the same conditional statement (such as E2 below) and others are logically different from, but truth functionally equivalent to, the basic conditional statement (such as E5 below). These equivalencies may be used with valid and invalid response options.

	Statement	**Equivalence**
EQ1	if p then q	p only if q
EQ2	if p then q	q if p
EQ3	if p then q	not p unless q
EQ4	if p then q	not (both p and not-q)
EQ5	if p then q	either not-p or q

Table S: Two Statements Connected; p and q

S	Premise	if p then q
S1	Valid Conclusion	if p, then q
S2		if non-q, then non-p
S3	Invalid Conclusion	if p then non-q
S4		if non-p then q
S5		if non-p then non-q*
S6		if q then p*
S7		if q then non-p
S8		if non-q then p

*Illogical Bias

Table T: Two Statements Connected; p and q

T	Premise	p if and only if q
T1	Valid Conclusion	p if and only if q
T2		non-p if and only if non-q
T3		q if and only if p
T4		non-q if and only if non-p
T5		if p, then q
T6		if non-q, then non-p
T7		if q, then p
T8		if non-p, then non-q
T9	Invalid Conclusion	p if and only if non-q
T10		non-p if and only if q
T11		q if and only if non-p
T12		non-q if and only if p
T13		if p, then non-q
T14		if non-p, then q
T15		if q, then non-p
T16		if non-q, then p

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Part D: Deductive Reasoning with Three Connected Statements: Table RS

In Part D, a table is given showing the valid and invalid conclusions for a syllogism based on two connected statements. Each premise is a complex statement containing two statements, and each conclusion is complex statement containing two statements. The two premises have one statement in common, denoted by "r." The other two statements in the premises are denoted by "p" and "q" as shown in the table.

Note: The equivalencies of the conditional statement apply here also.

	Statement	**Equivalence**
EQ1	if p then q	p only if q
EQ2	if p then q	q if p
EQ3	if p then q	not p unless q
EQ4	if p then q	not (both p and non-q)
EQ5	if p then q	either non-p or q

Table RS: Three Statements Connected; p, q, and r

	Premise	if r then q
	Premise	if p then r
RS1	Valid Conclusion	if p, then q
RS2		if non-q, then non-p
RS3	Invalid Conclusion	if p then non-q
RS4		if non-p then q
RS5		if non-p then non-q*
RS6		if q then p*
RS7		if q then non-p
RS8		if non-q then p

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Part E: Inductive Reasoning with Two Sets: Tables B, C, F and G

In Part E, four tables are given showing the valid and invalid conclusions based on two-set premises. The first set of the initial premise is denoted by "S," and the second set is denoted by "P."

Table B:
Two Premises with Two Sets and the Quantifier in the Initial Premise

B	Premises	Of all S, m/n are P. a is an S
B1	Valid Conclusion	with a probability of m/n, a is a P
B2		with a probability of 1 - m/n, a is not a P
B3		with a probability of m/n, a is not a non-P
B4		with a probability of 1 - m/n, a is non-P
B5	Invalid Conclusion	with an unknown probability, a is a P
B6		with an unknown probability, a is not a P
B7		with a probability of m/n, a is not a P
B8		with a probability of 1 - m/n, a is a P
B9		with an unknown probability, a is not a non-P
B10		with an unknown probability, a is a non-P
B11		with a probability of m/n, a is a non-P
B12		with a probability of 1 - m/n, a is not a non-P

Table C:
Two Premises with Two Sets and the Quantifier in the Initial Premise

C	Premises	Of all S, m/n are P. a is an P
C1	Valid Conclusion	with an unknown probability, a is an S
C2		with an unknown probability, a is not an S
C3		with an unknown probability, a is not a non-S
C4		with an unknown probability, a is a non-S
C5	Invalid Conclusion	with a probability of m/n, a is an S*
C6		with a probability of m/n, a is not an S
C7		with a probability of m/n, a is a non-S
C8		with a probability of m/n, a is not a non-S*
C9		with a probability of 1 - m/n, a is an S
C10		with a probability of 1 - m/n, a is not an S
C11		with a probability of 1 - m/n, a is a non-S
C12		with a probability of 1 - m/n, a is not a non-S

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Table F:
Two Premises with Two Sets and the Quantifier in the Initial Premise

F	Premises	Of all S, m/n are not P. a is an S
F1	Valid Conclusion	with a probability of m/n, a is not a P
F2		with a probability of 1 - m/n, a is a P
F3		with a probability of m/n, a is a non-P
F4		with a probability of 1 - m/n, a is not a non-P
F5	Invalid Conclusion	with an unknown probability, a is not a P
F6		with an unknown probability, a is a P
F7		with a probability of m/n, a is a P
F8		with a probability of 1 - m/n, a is not a P
F9		with an unknown probability, a is a non-P
F10		with an unknown probability, a is not a non-P
F11		with a probability of m/n, a is not a non-P
F12		with a probability of 1 - m/n, a is a non-P

Table G:
Two Premises with Two Sets and the Quantifier in the Initial Premise

G	Premises	Of all S, m/n are not P. a is an P
G1	Valid Conclusion	with an unknown probability, a is not an S
G2		with an unknown probability, a is an S
G3		with an unknown probability, a is a non-S
G4		with an unknown probability, a is not a non-S
G5	Invalid Conclusion	with a probability of m/n, a is not an S*
G6		with a probability of m/n, a is an S
G7		with a probability of m/n, a is not a non-S
G8		with a probability of m/n, a is a non-S*
G9		with a probability of 1 - m/n, a is not a S
G10		with a probability of 1 - m/n, a is an S
G11		with a probability of 1 - m/n, a is not a non-S
G12		with a probability of 1 - m/n, a is a non-S

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Part F: Inductive Reasoning with Two Connected Statements: Tables V and W

In Part F, two tables are given showing the valid and invalid conclusions based on connected statements. The first statement of the initial premise is denoted by "p," and the second statement is denoted by "q."

Table V:
Two Premises with Connected Statements and the Probability in the Initial Premise

V	Premises	if p, then q, with a probability of m/n. p
V1	Valid Conclusion	with a probability of m/n, q
V2		with a probability of 1 - m/n, not q
V3	Invalid Conclusion	with an unknown probability, q
V4		with an unknown probability, not q
V5		with a probability of m/n, not q
V6		with a probability of 1 - m/n, q

Table W:
Two Premises with Connected Statements and the Probability in the Initial Premise

W	Premises	if p, then q, with a probability of m/n. q
W1	Valid Conclusion	with an unknown probability, p
W2		with an unknown probability, not p
W3	Invalid Conclusion	with a probability of m/n, p*
W4		with a probability of m/n, not p
W5		with a probability of 1 - m/n, p
W6		with a probability of 1 - m/n, not p

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